

EJERCICIO 2

2

$X = \text{"n: teléfonos móviles"}$
(u.a. discreta)

. F. Probabilidad ;

x	1	2
$f_x(x)$	$\frac{3}{4}$	$\frac{1}{4}$

$\rightarrow \Sigma = 1$

a) $E(X)$ y $Var(X)$

$$E(X) = 1 \cdot \frac{3}{4} + 2 \cdot \frac{1}{4} = \frac{5}{4} = 1'25 \text{ teléfonos.}$$

$$Var(X) = \overline{E(X^2)} - E^2(X) = 1'75 - 1'25^2 = 0'1875 \text{ teléfonos}^2$$

$$E(X^2) = 1^2 \cdot \frac{3}{4} + 2^2 \cdot \frac{1}{4} = 1'75 \text{ teléfonos}^2.$$

b) m.a.s. tamaño $n = 3$

b1)

X_1	X_2	X_3
1	1	1
1	1	2
1	2	1
2	1	1
1	2	2
2	1	2
1	2	2
2	2	2

(*)

Prob.

$27/64$

$9/64$

$9/64$

$9/64$

$3/64$

$3/64$

$3/64$

$1/64$

1

b2)

$\bar{X}$

1

$4/3$

$4/3$

$4/3$

$5/3$

$5/3$

$5/3$

2

Universidad

$$(*) \quad P(X_1 = 1, X_2 = 1, X_3 = 1) =$$

$$= \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$$

$$P(X_1 = 1, X_2 = 1, X_3 = 2) =$$

$$= \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{9}{64}$$

⋮

$$P(X_1 = 2, X_2 = 2, X_3 = 1) =$$

$$= \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{64} = \frac{3}{64}$$

⋮

$$P(X_1 = 2, X_2 = 2, X_3 = 1) =$$

$$= \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{64} = \frac{3}{64}$$

⋮

$$P(X_1 = 2, X_2 = 2, X_3 = 2) =$$

$$= \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64}$$

5

b2) F. Prob. de $\bar{x}$

$$\bar{x} = \frac{x_1 + x_2 + x_3}{3} \rightarrow \text{ver b1)}$$

F. Probabilidad :

$\bar{x}$	1	$4/3$	$5/3$	2
$f(\bar{x})$	$27/64$	$27/64$	$9/64$	$1/64 \rightarrow Z=1$
		$\downarrow$	$\downarrow$	
		$3 \cdot \frac{9}{64}$	$3 \cdot \frac{3}{64}$	

b3) $E(\bar{x})$

$$\begin{aligned}
 E(\bar{x}) &= 1 \cdot \frac{27}{64} + \frac{4}{3} \cdot \frac{27}{64} + \frac{5}{3} \cdot \frac{27}{64} + \\
 &+ 2 \cdot \frac{1}{64} = 1'25 + 14 = E(x)
 \end{aligned}$$

b4) $\text{Var}(\bar{x})$

$$\begin{aligned}\text{Var}(\bar{x}) &= \overbrace{E(\bar{x}^2)} - E^2(\bar{x}) = \\ &= 1'6250 - 1'25^2 = \\ &= 0'0825 + ef^2 \quad (\neq \text{Var}(x))\end{aligned}$$

$$\begin{aligned}E(\bar{x}^2) &= 1^2 \cdot \frac{27}{64} + \left(\frac{4}{3}\right)^2 \cdot \frac{27}{64} + \\ &+ \left(\frac{5}{3}\right)^2 \cdot \frac{9}{64} + 2^2 \cdot \frac{1}{9} = \\ &= 1'6250 + ef^2.\end{aligned}$$

NOTA : $\text{Var}(\bar{x}) = \frac{\text{Var}(x)}{n}$

b5) $P(\bar{x} < \mu)$

$$\begin{aligned}P(\bar{x} < \mu) &= P(\bar{x} < 1'25) = \\ &= P(\bar{x} < 1) = \frac{27}{64}\end{aligned}$$